8

ON THE

LINES
THAT DIVIDE EACH SEMIDIURNAL ARC
INTO
SIX EQUAL PARTS.

BY

W. A. CADELL, Esq. F. R. S. Lond. & Ed.

FROM THE

TRANSACTIONS

OF THE

ROYAL SOCIETY OF EDINBURGH.

[*Read 3d June* 1816.]

EDINBURGH:
PRINTED BY NEILL & CO.
1816.

ON THE

LINES

THAT DIVIDE EACH SEMIDIURNAL ARC

INTO

SIX EQUAL PARTS.

THE divisions of the day which different nations have employed, are denoted by hour-lines of various kinds on the sphere. Of these hour-lines, drawn on a supposition that neglects the inequalities of the Earth's motion, there are three kinds.

The first kind denotes *hours counted from the meridian, equal to each other at all declinations of the sun.* These lines are *great circles* on the sphere, passing through the poles of the equator, and every pair intercepting a *similar arc* on each of the parallels. Of this kind are the hour-lines of sidereal time, counted from the meridian, and the hour-lines of solar time, counted from the meridian.

The second kind of hour-line denotes *hours counted from the horizon, equal to each other in duration at all declinations of the sun.* These lines *are great circles* which touch the greatest visible parallel on the one hand, and the greatest invisible parallel on the other; each pair of these great circles cuts off a *similar arc* from each diurnal arc. They are the horizons of different points of the parallel which passes through the zenith. The Italian hours counted from sunset, and the Babylonian hours counted from sunrise, are denoted by lines of this kind.

The third kind of hour-line, and of which it is proposed to speak more particularly here, denotes *hours varying in length as the declination of the sun varies*, each hour being one-sixth part of the semidiurnal arc, whether that arc be a smaller portion of the circumference, as in winter, or a greater, as it is in summer. On the oblique sphere these lines are *not great circles*, and each adjacent pair intercepts *a dissimilar arc* on each semidiurnal arc. This kind comprehends the hour-lines of the ancient Greeks and Romans, which denote hours called *hectemoria* *, that is, sixth parts of the semidiurnal arc.

The curvature of these lines is visible when they are drawn on a globe; it is likewise seen in their gnomonic projection, in the following manner.

Figure 1st is a perspective view of the lines which intercept one-sixth part of each semidiurnal arc; the point of sight is the centre of the sphere; the plane of projection touches the sphere at the pole of the equator, and is therefore parallel to the equator; the latitude is 66° 30′; at this latitude the whole of each hectemorial hour-line is gone over by the sun in a year. This perspective view is the same as the central or gnomonic projection of the sphere on the inside of a plane which touches the sphere at the pole of the equator; it forms an inferior equinoxial dial for the latitude 66° 30′, when placed parallel to the equator, with its inscribed surface downwards, and the point xxiv. elevated.

---

* Ἑκτημόριον, sexta pars, sextarius, is used by PTOLOMY. The lines that separate the hectemoria from each other are in this paper called *hectemorial lines*.

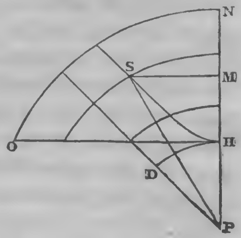
In this view the projections of the parallels are circles, and equal arcs of the parallels are represented by equal arcs of their projections: the construction of the lines which intercept one-sixth part of each semidiurnal arc, is therefore performed by dividing the projection of each semidiurnal arc into six equal parts, and connecting each point of division with its corresponding points on the projections of the other semidiurnal arcs.

In figure 1st, these hectemorial hour-lines are seen to converge at that point of the meridian which is marked 66° 30′. This is the point of contact of the horizon, and greatest unseen parallel; it is also the point where the mid-day part of the meridian cuts the horizon. At this point the semidiurnal arc is indefinitely small, and therefore the lines which divide it into six parts must be indefinitely near to each other, or, in other words, must converge.

At this point of convergence each hectemorial line is inclined at a considerable angle to the meridian. As the line proceeds, the inclination becomes less, till it is nearly as small as the inclination of the astronomical hour-line, which this hectemorial line cuts at the equator; and the hectemorial line on this projection is asymptotic with that astronomical hour-line. For the distance of their intersection from P, measured on the plane of projection, is infinite, being equal to the distance of the intersection of the equator and plane of projection, two planes parallel to each other; and however far the projection is extended, the two lines approach indefinitely, but do not meet.

Take, for example, the third hectemorial line HS (in the figure on the margin) which cuts the ninth astronomical hour-line at the equator; the projection of a great circle which intersects the ninth astronomical hour-line at the equator, is a straight line parallel to the ninth astronomical hour-line on this projection; but the third hectemorial line is continually approaching to the ninth astronomical hour-line; the distance between them at the horizon being HD = *arc.* 45° × *tan. polar distance*, and afterwards it is (*arc* 45° — $\frac{1}{2}$ *semidiurn. arc*) × *tan. polar distance of the star*; $\frac{1}{2}$ *semid. arc* increases, but never attains to be 45°, so that the distance never becomes equal to nothing, and *tan. polar distance* increases indefinitely.

All great circles are seen under the form of straight lines in this projection of the sphere; and therefore the projection of one great circle cannot be an asymptot to the projection of another; it follows, that the projections of the hectemorial lines are not projections of great circles. If a straight line be drawn through the point H, (in the figure on the margin,) cutting off a given aliquot, one-half, for example, from a semidiurnal arc on the projection, it will cut off a smaller aliquot from the meridional extremity of the other semidiurnal arcs, in proportion as they are nearer to the point H; and in order that a straight line drawn from H may cut off the same aliquot part from several concentric arcs which are included between the versed sine HN and sine HO of the outer arc, it is necessary that the chords of these arcs be parallel to each other; which happens only in the case where all the arcs are of 90°, then H coincides with P, and then the straight line which cuts off the same aliquot from every arc, is a line passing through P the centre; and this sole case is a central projection of a sphere so placed, that each semidiurnal arc is 90°, the poles of the equator being in the horizon. In this position alone are the hectemorial



hectemorial lines great circles, and in this case they coincide with the astronomical hour-lines, and pass through the poles of the equator. When the poles are thus in the horizon, all the three kinds of hour-lines coincide. When the pole is in the zenith, the second kind or horizontal, and the third or hectemorial hour-lines cease to be, because then the horizon does not cut any of the parallels into diurnal and nocturnal arcs.

To express algebraically some of the above-mentioned properties; Let the abscissæ x be taken on the meridian HM, and the ordinates y at right angles to the meridian, the central projection gives

$$y = \textit{sin.}\ \frac{n}{6}\ s\ \textit{tan. polar dist. star},$$

$$x = \left\{ \textit{cos.}\ \frac{n}{6} s - \textit{cos.}\ s \right\} \textit{tan. pol. dist. star.}$$

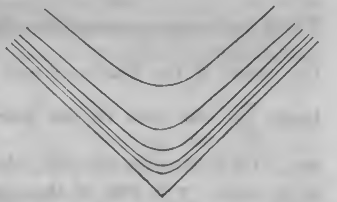
PH is $\textit{cos.}\ s$; the cosine of the semidiurnal arc. PM is $\textit{cos.}\ \frac{n}{6} s$; the cosine of the fractional part of the semidiurnal arc. MS is $\textit{sin.}\ \frac{n}{6} s$; the sine of the fractional part of the semidiurnal arc. PS is *tan. pol. dist. star*; the tangent of the polar distance of the star. n is one of the numbers 1, 2, 3, 4, 5; it is 5 for the curve that contains the first and eleventh hectemorial line; 4 for the second and tenth, and so forth.

$\frac{y}{x} = \frac{\textit{sin.}\ \frac{n}{6}}{\textit{cos.}\ \frac{n}{6} s - \textit{cos.}\ s}$ is not the equation of a straight line, except in the case where $\textit{cos.}\ s = 0$, that is, when the semidiurnal arcs are all 90°, the poles being in the horizon.

$y = sin. \frac{n}{6} s\ tan.\ pol.\ dist.$ in this equation, when the describing diameter arrives at H, $sin. \frac{n}{6} s = 0$, that is, the curve cuts the axis of the abscissæ at H, and when the describing diameter passes onward, the sign of $sin. \frac{n}{6} s$ is changed, expressing that there are two similar and equal branches, one on each side of the axis of the abscissæ.

$Cos. \frac{n}{6} s$ has only one value, because $\frac{n}{6} s$ is never so great as 90°, and its cosine, therefore, does not become $= 0$, which it must do before its sign changes.

If a central projection of any one forenoon hectemorial hour-line, and of the afternoon hectemorial line equi-distant from the meridian, be drawn for different heights of the pole, it will be seen that these two hectemorial lines form for each height of the pole an equicrural curvilinear branch, including within it the corresponding hectemorial lines for a higher latitude, and all of them included within two astronomical hour-lines as asymptots. With these asymptots the pair of hectemorial lines coincides, when the poles are in the horizon. The figure on the margin represents the equicrural curves formed by the third hectemorial hour-line and by the ninth, which is the afternoon branch equidistant from the meridian. The curved branches are drawn for every ten degrees of latitude from 30 to 70 inclusive; the rectilinear asymptots within



in which the curves are contained, are the ninth and fifteenth astronomical hour-line counted from midnight.

From figure 1st it may be collected, that the asymptots of each hectemorial line are as follows :

The equicrural curvilinear branch, composed of the hectemorial lines	*has for assymptots the astronomical hour lines, counted from midnight,*	*which comprehend an angle of*
First A, and eleventh IA,	VII. and XVII.	150°
Second B, and tenth I,	VIII. and XVI.	120
Third Γ, and ninth Θ,	IX. and XV.	90°
Fourth Δ, and eighth H,	X. and XIV.	60°
Fifth, E, and seventh Z,	XI. and XIII.	30°

Figure 1st represents the portions of the day hectemorial lines which touch the greatest invisible parallel for the latitude 66° 30′; these are the winter portions. In order to delineate the portions which touch the greatest wholly visible parallel, or the summer portions, the projection is made on the plane which touches the sphere at the depressed pole; this projection is exhibited in figure 2d, which contains portions of curves, each of which is similar and equal to the curve of the same hour in figure 1st, but differently placed. It forms a superior equinoxial dial for 66° 30′, when placed parallel to the equator, with the inscribed surface upwards, and the point xxiv. elevated.

However extended the plane of projection touching the sphere at the pole be, still it will not contain the portions of the hectemorial lines that are near the equator. To have these, the plane of projection is taken at right angles to the equator, and touching the sphere at the depressed intersection of the equator and meridian. This projection, which is drawn at figure 3d, contains some of the hectemorial lines from the point of their contact with the greatest wholly unseen to the points of their contact with the greatest wholly seen parallel. It forms a polar dial for the latitude 66° 30′, when placed parallel to the plane of the sixth astronomical hour-line.

The lines thus drawn in the above projections are curved; consequently they are not the projections of great circles; neither are they the projections of small circles, for if they were, they must necessarily touch the horizon at its intersection with the mid-day portion of the meridian, because in that point the hectemorial lines converge, and do not go on the other side of the horizon. Now, if small circles so placed, be drawn on the sphere or projected on a plane, it will be found that their course deviates entirely from the course of the lines bounding the hectemoria. The hectemorial lines, therefore, do not coincide with small circles on the sphere, nor with conic sections on the central projection.

The projections above exhibited shew that each pair of hectemorial lines for a given meridian and latitude, is an equicrural curved line; but this is only one branch of an entire curve, because the diameter whose extremity has traced a pair of these lines on the surface of the sphere, has still to complete its revolution, which is done when it has arrived by progressive and continuous motion at the point from which it set out. In order to accomplish this with the same kind of motion with which they described the first branch, the extremities of the diameter leave the two parallels that touch the horizon, and proceed to cut off the same aliquot part from the semidiurnal arcs belonging to this second point of departure, as they had done from the semidiurnal arcs of the first point of departure; the second point of departure is to be considered as the mid-day point of a horizon on the opposite side of the equator to the first. The extremities of the diameter, after having in this way passed several times to and fro between the greatest wholly unseen and the greatest wholly seen parallel, and after having completed one or more circumferences, attain precisely the two opposite points from which they set out, having formed two opposite re-entering curves on the surface of the sphere, and afterwards, in every subsequent revolution, the extremities of the diameter only retrace the lines they had described in their first revolution. The nature of the motion is such, that the describing points cannot go beyond the two parallels which touch the horizon, because there are no semidiurnal arcs beyond these parallels, and the constitution of the hectemorial lines, consists in cutting the semidiurnal arcs.

Whilst the extremities of the diameter describe two re-entering curves on the surface of the sphere, the diameter itself describes two opposite re-entering curved surfaces, whose common vertex is the centre of the sphere. These two opposite surfaces coincide with a straight line directed to the vertex, but do not coincide with a straight line in any other direction. In this respect they resemble a conical surface, and they may be considered as a kind of opposite cones with an undulated surface, included between the outer surfaces of the two opposite circular based cones that touch the horizon; the apex of each undulation being applied alternately to the one and to the other of these right circular based cones, so that as the common vertex of all these cones is the centre of the sphere, each of the two opposite undulated cones has two similar and equal bases or right sections, the one above the centre of the sphere, the other below it. Each of these bases is an uninclosed curve, composed of many equicrural branches, each equicrural branch having for asymptots two straight lines that intersect in the axis; these two right sections are indefinitely extended; and the bicrural branches of one right section are alternately placed in respect to the branches of the other right section.

The undulated cone belonging to each of the five hectemorial lines is different from that of the others. The five upper bases are represented in figures 4th, 5th, 6th, 7th, and 8th, drawn according to the rules of the central projection, on a plane parallel to the equator, for the latitude 66° 30′. The full lines are the upper base of one of the two opposite undulated cones; the dotted lines are the upper base of the other opposite undulated cone. These bases or right sections are each made up of a pair of hectemorial lines for a given latitude, forming one equicrural branch, and of similar and equal pairs for other points of the same parallel of latitude and of the opposite parallel.

In order to present to the eye the image of one of the undulated cones, figure 9th is drawn. It is a shaded view of one of the two opposite conical surfaces to which the 3d and 9th hectemorial lines belong, for the latitude 66° 30′. The point of sight is in the plane of the equator; the distance is two diameters from the centre of the sphere.

The sections of the undulated cones by a cylindrical surface circumscribed round the equator, are made up of the same branches; they are complete and re-entering, whereas the sections by a plane are always incomplete. These cylindrical sections are laid down in figures 10th, 11th, 12th, 13th, and 14th. The cylindrical surface is here unrolled; when restored to its cylindrical form, the inscribed curve is re-entering, and without a break.

Where the number of degrees between the upper apices of each undulation is a divisor of 360, the undulated cone is completed in one circumference, because at the beginning of the second circumference, the generating diameter enters into the path it had described in the first; this is the case with the three undulated cones which contain the third, fourth and fifth hectemorial lines, figures 6th and 12th, 7th and 13th, 8th and 14th.

The number of degrees between the meridians of two adjacent upper apices, is equal to twice the distance between the two adjacent sides of each undulation measured on the equator, or to four times the distance of the equatorial point of the hectemorial line from the meridian measured on the equator.

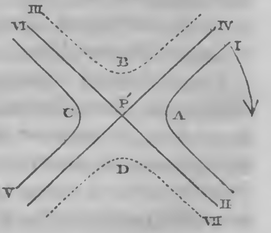
Where this number of degrees between the meridians of the upper apices is not a divisor of 360, but of a multiple of 360, the revolution is completed in as many circumferences as there are units in that multiple; because the generating diameter does not enter into its former path till it has described that number of circumferences. The undulated cone which contains the second and tenth hectemorial lines, having an interval of 16 equinoxial hours, or 240° between the adjacent upper apices; the describing diameter completes its revolution, and comes to the point from which it set out at the end of two circumferences or 720°; the number of the upper apices being 3, which, multiplied by 240, is equal to 360 × 2; this is seen in figures 5th and 11th.

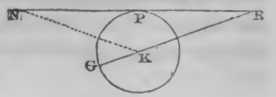
The undulated cone which contains the first and eleventh hectemorial lines, figures 4th and 10th, having an interval of 20 equinoxial hours, or 300° between the upper apices of the undulations, the describing diameter must go over five circumferences before it comes to the point from which it set out, because 300 multiplied by 6, which is the number of upper apices, is equal to 360 × 5.

The circumstances of each of the undulated cones with respect to the number of undulations and of circumferences, are as follows:

The undulated cone containing the hectemorial lines	*intercepts between two adjacent upper apices,*		*The number of upper apices is*	*The number of circumferences in which a revolution is completed, is,*
	In degrees.	*In equinoctial hours.*		
First A, and eleventh IA,	300	20	6	5
Second B, and tenth I,	240	16	3	2
Third Γ, and ninth Θ,	180	12	2	1
Fourth Δ, and eighth H,	120	8	3	1
Fifth E, and seventh Z,	60	4	6	1

Four of the five undulated cones have opposite and similar undulated cones, formed by the remote half of the generating diameter; but in that undulated cone which contains the fourth hectemorial line Δ, and eighth H, the two opposite undulated cones coincide in one, as is seen in figures 7th and 13th.





The algebraic formula expresses the different branches of the right section of the undulated cone, by the change of sign of *tan. pol. dist.* in the values of x and y;

$$x = \left(\cos\frac{n}{6}s - \cos s\right)\tan.\ pol.\ dist.$$

$$y = \sin.\ \frac{n}{6}s\ \tan.\ pol.\ dist.$$

For example, in the right section of the undulated cone which contains the third and ninth hectemorial lines, the radii from the centre P′ are = *tan. pol. dist.*, and the co-ordinates are affected by that quantity. If the generating diameter GKR′ move in the direction of the dart, and set out from I; it describes the curvilinear branch A, and *tan. pol. dist.* is positive till the generating diameter come to the situation II, where it is infinite. If its revolution be continued, it re-appears at III, on the other side of the centre with the negative sign, and gives the curvilinear branch B; at IV. it is again infinite, and re-appears positive at V, going on to form the curvilinear branch C; at VI. it is infinite; and at VII. it becomes negative, and forms the branch D; it then comes out positive at I, and goes over its former path. The way that the change of sign takes place is apparent, by considering that whilst the diameter is moving in the direction of the dart, it has another motion at right angles to the plane of projection. Let GPK be a section of the generating sphere, at right angles to the plane of projection; NPR being the section of that plane; GKR is the generating diameter; and when in the position GKR, then *tan. pol. dist.* = PR, and is positive; when GK is parallel to PR, then *tan. pol. dist.* is infinite. If the motion in this plane be continued, *tan. pol. dist.* passes to the other side of P, as at PN, and is affected with the contrary sign.

Most of the writers who have spoken of the hectemorial lines, have treated them as great circles, because their intertropical parts, at a moderate height of the pole, coincide sensibly with great circles; and it is this case with which authors had to do in considering the gnomonic projection of the hectemorial lines for the climates of Greece or of Italy. The writings of a considerable number of authors on this subject have been consulted, and they all take the hectemorial lines for great circles, except CLAVIUS and MONTUCLA. CLAVIUS demonstrates that the antique hour-lines do not coincide with great circles;

and Montucla merely states, but without discussion, that they are curves of a peculiar nature *.

It has been shewn above, chiefly by means of a projection on a plane touching the sphere at the pole, that the hectemorial hour-lines on the oblique sphere are not great circles ; and because the describing diameter, in order to form a continuous and uniform surface, must go on moving during its whole revolution with that motion which it had in the beginning of its course, and must be always included between the two parallels that touch the horizon, it is concluded that the curved surface

whose

---

* The passages from Clavius and Montucla are as follows:

Clavii Astrolabium lib. i. lemma 39 " Circuli maximi transeuntes per horas inæquales Æquatoris, et duorum parallelorum oppositorum, non necessario per horas inæquales parallelorum intermediorum transeunt in sphæra obliqua." He gives a demonstration of this, and concludes, in the *scholium*, that in order to delineate the antique hours with strict accuracy, a considerable number of the semidiurnal arcs are to be divided into six parts, and the corresponding points of division joined.

Montucla, Hist. des Math. tom i. edition de 1758 : " Les lignes de ces sortes d'heures [les heures antiques] ne sont point droites comme les precedentes, mais courbes, et meme d'une forme tres bizarre ; de sorte qu'on ne peut les decrire qu'en determinant plusieurs points de chacune ; la manière de les trouver se presentera facilement à tout geometre ; c'est pourquoi nous ne nous y arretons pas."

The circumstance mentioned in the beginning of the paragraph to which this note refers, has led the celebrated and profound astronomer Delambre to controvert the opinion of Montucla in the following words : " Montucla dit, en parlant des heures temporaires antiques, qu'elles sont courbes, et même d'une forme très bizarre, &c. *Hist. des Mathem.* tom i. On ne conçoit pas comment une pareille inadvertance a pu echapper à un homme aussi instruit ; car si la surface est spherique, ces lignes seront des grands cercles ; et si la surface est plane, elles seront des lignes droites, puisqu'elles seront les intersections des plans de ces grands cercles avec le plan du cadran." Delambre *sur un cadran antique trouvé dans l'isle de Delos, et par occasion de la gnomonique des anciens ; notice lue à la classe des Sciences Physiques et Mathematiques de l'Institut Royal de France, le* 10 *Octobre* 1814.

whose section contains the hectemorial line, is a kind of undulated conical surface; and the right sections of the surface are two infinite unenclosed bases, one on each side of the vertex of the undulated cone. This section, at right angles to the axis, consists of several bicrural branches, varying in number as the cone belongs to each different hectemorial line.

Of the Gnomonic Instruments of the Ancients.

The object of the preceding pages has been, to treat of the curves to which the hectemorial lines belong. As an appendix, it may not be improper to enumerate some of the remains of art which contain the antique hour lines; for these hour lines are the intertropical parts of the hectemorial lines. Several examples of these gnomonic instruments exist.

The first to be mentioned, and the most perfect, are the eight sun-dials on the Tower of Andronicus Cyrrhestes, at Athens. They appear to have been coeval with the building, and to have formed part of the original design, as may be inferred from the care with which they are delineated, and from the greatest part of the surface of the wall being left plane to receive the lines. This tower is mentioned under the denomination of *horologium* by VARRO, who flourished in the 85th year before the Christian era; it is also spoken of by VITRUVIUS. The carefully wrought channels, and cylindrical cavities in the pavement, and the cylindrical chamber at the south side, have led to the conjecture, that, besides serving to shew the hour when the sun was shining on it, the tower was formed to contain some machine of the nature of the clepsydra, whereby the hour might be known at all times for the use of the city. Another of the destinations of this tower was to indicate the direction of the wind. Each of the eight dials is exposed to one of the eight principal equidistant points of the horizon; two of the dials being parallel to the plane of the meridian. The radii of the spheres from which the dials are projected, vary; the smallest being about eight inches, and the greatest about twenty-five inches. In STUART's *Antiquities of Athens*, the building and the dials are represented in detail.

The second example of the gnomonic lines of the ancients, is in the valuable collection of antiquities brought from Athens by the Earl of Elgin. It consists of four vertical dials, two of which are nearly south-east, and two nearly south-west in azimuth. They are inscribed on a block of white marble, which bears the maker's name. The radius of the generating sphere is about four inches and a half.

The third example, is a projection of the antique hour-lines on the inner surface of a cone whose axis is parallel to the axis of the earth. It exists at Athens, and is figured by STUART.

The fourth is a small east dial, on a vertical plane, described by DELAMBRÉ. It was found at Delos. The radius is half an inch.

The fifth is a piece of Roman workmanship, figured by BOISSARD*. It is composed of five dials on the upper part of a squared block of marble; three of the vertical sides of which are covered with an ancient Roman agricultural calendar. In like manner, the treatise of PALLADIUS *de re Rustica*, which is a set of agricultural directions for every month, contains a gnomonic table, shewing the length of a man's shadow for each month, and for a climate in Italy.

---

* BOISSARDI Antiquitates Romanæ.

Some fragments of ancient dials are also published by GRÆVIUS *.

Except the table of PALLADIUS, the instruments above mentioned are each made for an unvarying azimuth. In the description of the antiquities of Herculaneum †, there is represented and explained a dial whose azimuth is changeable, to suit the hour and the different declinations of the sun. It is drawn on an irregularly curved surface of bronze, and the declinations are marked with the Roman names of the months.

The ancient names of different kinds of fixed and moveable sun-dials, with the names of their inventors, are given by VITRUVIUS.

Something respecting the time of the first introduction of gnomonic instruments, is to be collected from Greek and Roman authors. The ancient inhabitants of Egypt appear to have cultivated astronomy at a time prior to the earliest historical accounts; and they have left a monument of their practical skill in that science, in the accurate meridional position of these most ancient of human works the Pyramids. Portions of their knowledge were diffused amongst the Hebrews and Babylonians.

The astronomical science of the Greeks was derived partly from the Egyptians, and partly from the Babylonians ‡. THALES ‖ acquired his knowledge of astronomy and geometry from the Egyptian priests, and introduced these sciences into Greece. The gnomonic projection of the sphere is a branch of the doctrine of spherical astronomy, and when applied to the

---

* GRÆVII Thesaurus Antiquitatum Romanarum.

† Le Pitture antiche d'Erculano, tom. iii. Napoli 1762.

‡ HERODOTUS.

‖ DIOGENES LAERTIUS.

the purpose of shewing the parts of the day, constitutes the Sun-dial, an instrument which may be supposed to have come into use soon after the introduction of astronomy. According to DIOGENES LAERTIUS and PLINY, the first gnomonic instrument that appeared in Greece was constructed by a disciple of THALES, about the 545th year before the Christian era, and 115 years before the death of PERICLES. SAUMAISE * contends, that this instrument was for the use of astronomers only, and that sun-dials did not come into general use in Greece till 200 years after, that is, a short time before the age of ALEXANDER.

The Romans got their first sun-dial from one of the Greek cities of Sicily †, 260 years before the Christian era; and for nearly a hundred years after, they possessed no artist acquainted with the principles of the instrument, so as to construct one adjusted to the climate of Rome.

---

* SALMASII Plinianæ Exercitationes.

† PLINII Hist. Naturalis.

Explanation

Explanation of the Figures.

Central Projection of the twelve Hour-lines, figure 1*st to figure* 3*d.*

Figure 1*st*, Is a central or gnomonic projection of the hectemorial lines or antique hour-lines, on the inside of a plane touching the sphere at the elevated pole of the equator. This and all the other figures are drawn for the latitude 66° 30′. Each antique hour-line is marked with the Greek numeral that belongs to it.

Figure 2*d*, Is a central projection of the summer portion of these lines on the inside of a plane touching the sphere at the depressed pole of the equator.

The circle placed between figure 1st and figure 2d is a great circle of that sphere from which all the projections in these figures (except figure 9th) are formed.

Figure 3*d*, Is a central projection of the hectemorial lines on a plane touching the sphere at the depressed intersection of the meridian and the equator.

Sections of the five undulated Cones which contain the antique hour-lines, by a plane at right angles to the axis, figure 4*th to figure* 8*th.*

Figure 4*th*, Is a section at right angles to the axis of the undulated conical surface, which contains the first antique hour-line A, and eleventh IA. In this and the other figures, the full lines are the section of one of the two opposite cones, and the dotted lines are the section of the other. The section of the same undulated cone by a cylindrical surface, is seen in figure 10th.

Figure 5th, Is the right section of the undulated cone which contains the second antique hour-line B, and tenth I. The section, by a cylindrical surface, is seen at figure 11th.

Figure 6th, Is the right section of the undulated cone which contains the third antique hour-line Γ, and ninth Θ. Its section, by a cylindrical surface, is figure 12th. A perspective and shaded view of this undulated conical surface is figure 9.

Figure 7th, Is the right section of the undulated cone which contains the fourth antique hour-line Δ, and the eighth H; figure 13th is its section by a cylindrical surface.

Figure 8th, Is the right section of the undulated cone which contains the fifth antique hour-line E, and the seventh Z. Figure 14. is its section by a cylindrical surface.

Perspective and shaded view of one of the Surfaces, figure 9th.

Figure 9th, Is a perspective and shaded view of the undulated conical surface which contains the third antique hour-line Γ, and ninth Θ; the point of sight is in the plane of the equator, and the distance of the eye is two diameters from the centre of the sphere; in this view only one of the two opposite surfaces is drawn. The right section of these opposite cones is figure 6th, and their section by a cylindrical surface is figure 12th.

Sections

Sections of the five undulated Conical Surfaces by a Cylindrical Surface, figures 10*th to* 14*th.*

Figure 10*th*, Is the section of the undulated cone which contains the first antique hour-line A, and eleventh IA, by a cylindrical surface touching the sphere at the equator. To avoid intricacy in this figure, the section of only one of the two opposite cones is drawn. Figure 4th is the right section of the two opposite surfaces.

Figure 11*th*, Is the section of the undulated cone which contains the second antique hour-line B, and tenth I, by a cylindrical surface. The full line is the section of one of the two opposite cones; the dotted line is the section of the other. Fig. 5. is the right section of this surface.

Figure 12*th*, Is the section of the two opposite surfaces which contain the third antique hour-line Γ, and ninth Θ, by a cylindrical surface. Their right section is figure 6th; and one of these two oppposite surfaces is represented in perspective, and shaded at figure 9th.

Figure 13*th*, Is the section of the undulated cone which contains the fourth antique hour-line Δ, and eighth H, by a cylindrical surface. In this undulated cone, the two opposite conical surfaces coincide in one: there are therefore no dotted lines, neither on this figure, nor on figure 7th, which is the right section of this undulated cone.

Figure 14*th*, Is the section of the undulated cone which contains the fifth antique hour-line E, and seventh Z, by a cylindrical surface. Figure 8th, is the right section of the same.

PLATE II.

Engraved for the Royal Society Trans.s Vol 8, page 82.

Fig. 1.

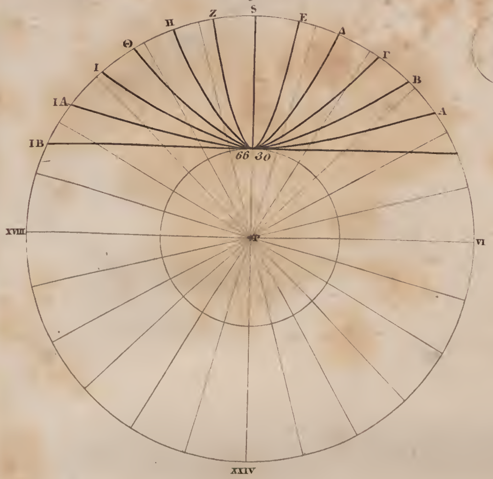


Fig. 2.

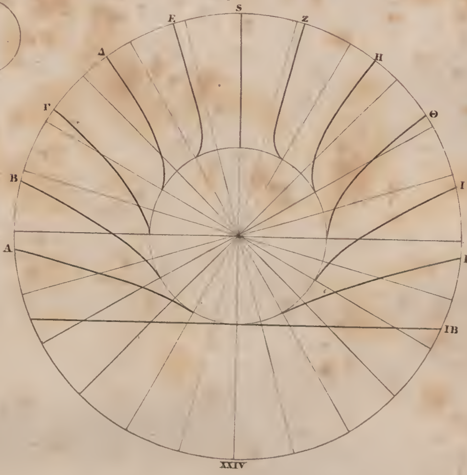


Fig. 3.

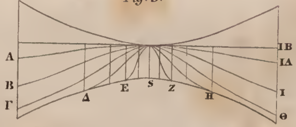


Fig. 4.

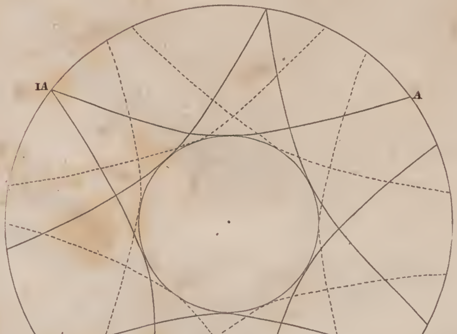
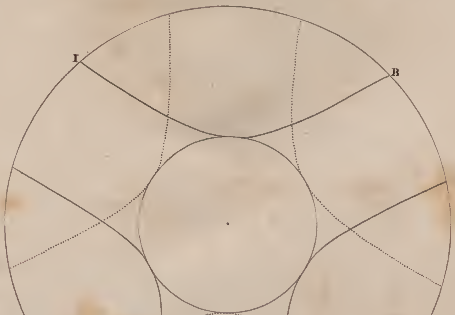


Fig. 5.



Engraved by W & D. Lizars Edin.r

PLATE III.

Engraved for the Royal Society Trans.s Vol. 8, page 82.

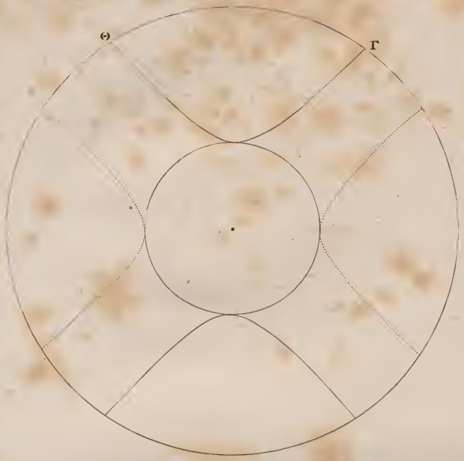


Fig. 6.

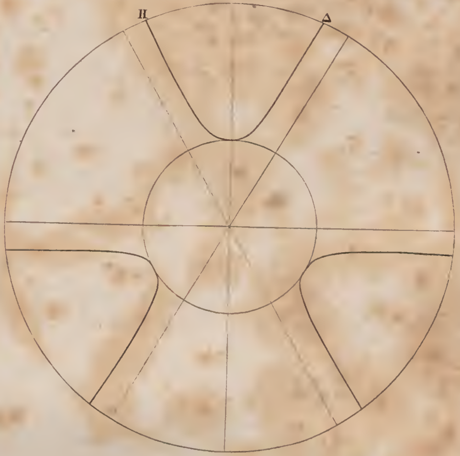


Fig. 7.

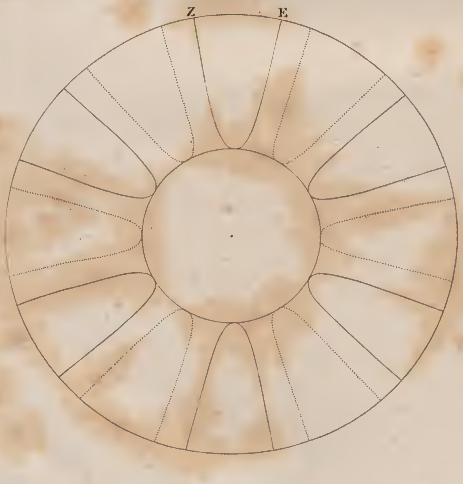


Fig. 8.

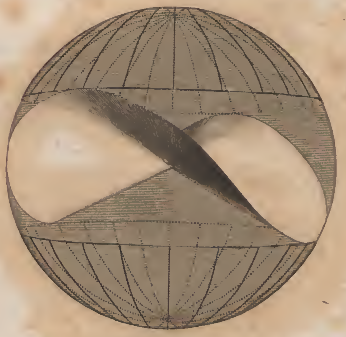


Fig. 9.

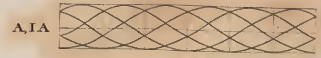


Fig. 10.

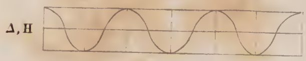


Fig. 13.

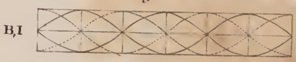


Fig. 11.

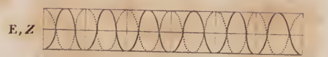


Fig. 14.

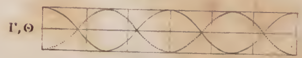


Fig. 12.

Engraved by W. & D. Lizars Edin.r